

# Engineering Notes

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## Extension of Flutter Prediction Parameter for Multimode Flutter Systems

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### Introduction

**A**EROELASTIC instability such as flutter involves aerodynamic, inertia, and elastic forces of flight vehicles.<sup>1,2</sup> If flutter occurs during the flight, the aircraft structure may fail or suffer serious damage. Therefore, it is important to predict accurate and reliable flutter boundary of a flight vehicle using flutter analyses and flight tests. Even when great care is taken to ensure the accuracy of the modeling techniques used, uncertainties in the aerodynamic modeling and structural modeling using aeroelastic analysis will inevitably be present. These uncertainties can result in inaccurate flutter predictions and potential failure of flight vehicles. Thus, to determine and correct the uncertainties in flutter analyses, wind-tunnel or flight tests must be conducted at the final stage of an aircraft design.

Generally, wind-tunnel and flight tests are conducted at a subcritical speed to avoid the failure or damage of a flight vehicle, and

then the flutter boundary is predicted using a stability criterion. Traditionally, this stability criterion is based on the modal damping.<sup>3,4</sup> However, it is not always possible to measure accurately modal damping due to noise and errors. For typical flutter, it is also difficult to predict an accurate flutter boundary because the damping decreases suddenly just before a critical speed. Thus, the flutter prediction method based on modal damping does not give a reliable prediction of an accurate flutter boundary, motivating alternative flutter boundary prediction methods.<sup>5–9</sup>

Zimmermann and Weissenburger<sup>5</sup> introduced a new stability criterion called the flutter margin, which is based on Routh's stability criteria and makes use of both the modal frequency and damping information. They showed the applicability of the flutter margin method for a binary flutter system. This method was later extended to a three-degree-of-freedom system by Price and Lee.<sup>6</sup>

Matsuzaki and Ando<sup>7</sup> introduced a new flutter prediction method based on Jury's stability analysis (see Ref. 10) and system identification method for discrete-time systems. Torii and Matsuzaki<sup>8</sup> defined a new stability index, called a flutter prediction parameter, that is calculated from the coefficients of an identified autoregressive moving average (ARMA) model. They showed that the new parameter is efficient for predicting the flutter boundary through numerical and experimental studies. However, the parameter proposed by Torii and Matsuzaki is only available to a binary flutter system. Because of the existence of control surfaces, wingtip launchers, and stores, real wing structures have more complex configurations than the wing model typically tested in a laboratory environment and have the flutter characteristics of higher modes. Therefore, this flutter prediction parameter should be extended to be applicable to multimode flutter systems.

Recently, Lind and Brenner<sup>9</sup> introduced the "flutterometer," which is quite different from the other approaches and is a model-based approach. This method is able to predict a reasonable flutter speed using low-speed test points, whereas the data-based approaches, like Zimmerman and Weissenburger's method and the ARMA method, may predict an inaccurate flutter speed using these low-speed points. Although data-based approaches do inaccurately predict the flutter speed with low-speed tests, they are able to predict accurately the flutter point using high-speed test points, whereas the model-based approach predicts a conservative flutter speed.

In the present study, the flutter prediction method proposed by Torii and Matsuzaki<sup>8</sup> is extended to predict the flutter boundary of a system with more than two modes. The characteristics of this updated parameter are investigated numerically and experimentally. The wind-tunnel tests for two types of wing models, a flat plate wing (clean wing) and a plate wing with a flap (flap-wing) are conducted to verify our predicted results. From the wind-tunnel test data, updated flutter prediction parameters are calculated, and the behavior and availability of the parameters are investigated. The flutter boundary is then predicted through a linear fitting of the parameter and is compared with several numerical methods.<sup>11,12</sup>

### Flutter Prediction Method

#### Review of Torii and Matsuzaki's<sup>8</sup> Flutter Prediction Parameter

A new flutter prediction method using system identification was proposed by Torii and Matsuzaki<sup>8</sup> for a binary flutter system. The method consists of an aeroelastic system identification and calculation of the flutter prediction parameter. The aeroelastic system is identified using a fourth-order ARMA model of the flutter test data

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and the flutter prediction parameter is calculated from the estimated autoregressive (AR) coefficients. The identified flutter prediction parameter can only be used for flutter systems with two predominant modes.

From the random response  $\{y(k)\}$ , the aeroelastic system can be represented accurately by an ARMA model.<sup>8</sup> The ARMA model consists of AR measurements and a moving average of white noise. The AR part is associated with the stability of the system, and the characteristic polynomial can be expressed by

$$G(z) = A_n(z - z_1) \cdots (z - z_n) \\ = A_n z^n + \cdots + A_1 z + A_0, \quad A_n > 0 \quad (1)$$

where  $z$  is the discrete-time variable,  $z_i, i = 1, \dots, n$ , are the complex conjugates roots of characteristic equation, and  $A_i$  are the coefficients of the characteristic polynomial. The order  $n$  of Eq. (1) should be set to  $2M$ , where  $M$  is the number of vibration modes considered.

From the determinant method in Ref. 10, the stability can be checked easily from the coefficients of the characteristic polynomial in Eq. (1) without solving for the characteristic roots. The stability parameter is defined as

$$F^-(n-1) = A_n^{n-1} \prod_{i < j} (1 - z_i z_j) = \det(X_{n-1} - Y_{n-1}) \quad (2)$$

where

$$X_k = \begin{bmatrix} A_n & \cdots & A_{n-k+1} \\ 0 & \ddots & \vdots \\ 0 & 0 & A_n \end{bmatrix}, \quad Y_k = \begin{bmatrix} A_{k-1} & \cdots & A_0 \\ \vdots & \ddots & 0 \\ A_0 & 0 & 0 \end{bmatrix} \quad (3)$$

This method defines the system to be unstable if  $F^-(n-1)$  is less than zero (Ref. 10).

For a binary flutter system, it is easy to calculate the value of  $F^-(3)$  from the coefficients of the characteristic polynomial of Eq. (1) using Eqs. (2) and (3). Though  $F^-(3)$  is a good index to predict the flutter dynamic pressure, its behavior is inferior. To overcome this inconvenience, Torii and Matsuzaki<sup>8</sup> proposed the following flutter prediction parameter for a binary flutter system:

$$F_Z = \frac{F^-(3)}{F^-(1)^2} = \frac{\det(X_3 - Y_3)}{(A_4 - A_0)^2} \quad (4)$$

From an analytic standpoint,<sup>8</sup> the flutter prediction parameter  $F_Z$  in Eq. (4) is similar to that of Zimmerman and Weissenburger<sup>5</sup> and is linearly related to the dynamic pressure. If the values of  $F_Z$  for dynamic pressure are calculated from the flight-test data, the flutter point can be obtained by the linear extrapolation of  $F_Z$ .

#### Updated and Generalized Flutter Prediction Parameter

The flutter prediction parameter  $F_Z$  is available to predict the flutter boundary of the two-mode (bending-torsion) flutter system. Although the parameter  $F_Z$  is not applicable for multimode system, this parameter can be used if higher modes are eliminated by a bandpass filter. However, if the higher modes are responsible for flutter or multimodes must be taken into account,  $F_Z$  cannot be used, and only  $F^-(n-1)$  is available to prediction the flutter boundary.

As stated earlier,  $F^-(n-1)$  itself is an inconvenient method for predicting the flutter boundary. Therefore, there is a need to define a new flutter prediction parameter that is linearly related to the dynamic pressure. To predict the flutter boundary of a multimode flutter system, the following parameter  $F_N$  is proposed:

$$F_N = F^-(n-1)/F^-(n-2)^2 \quad (5)$$

where  $F^-(n-1)$  and  $F^-(n-2)$  are defined by Eqs. (2) and (3).

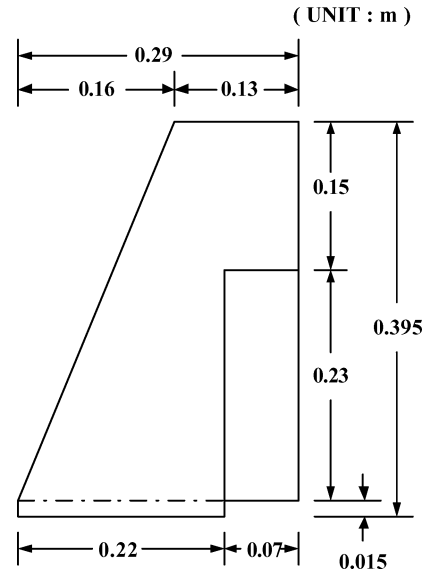


Fig. 1 Configurations of wind-tunnel test wing model.

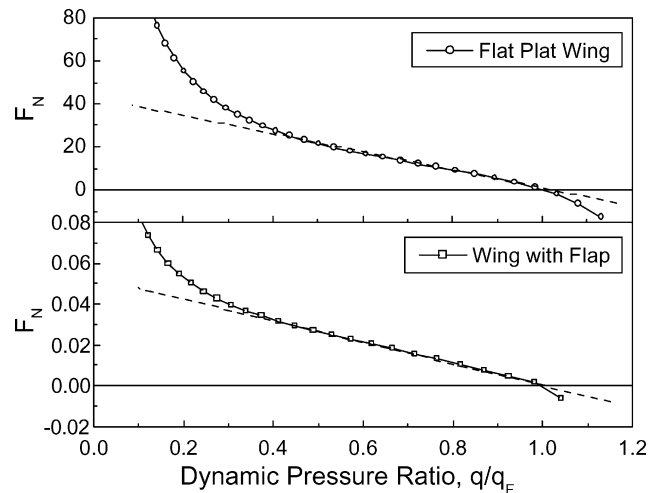


Fig. 2 Parameter  $F_N$  of flat plate wing model and wing with flap,  $n = 8$ .

#### Behavior of Updated Flutter Prediction Parameter

Although the present parameter  $F_N$  is similar to  $F_Z$  for a binary flutter system, it is necessary to check the linear relationship between  $F_N$  and the dynamic pressure. To investigate the behavior of the parameter  $F_N$ , it will be applied to a flat plate wing (clean wing) and wing with flap (flap-wing), shown in Fig. 1.

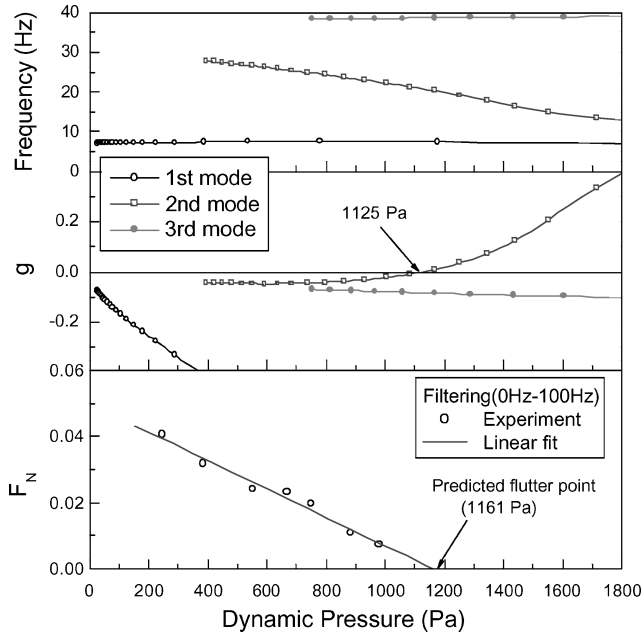
Figure 2 shows the variation of  $F_N$  for aeroelastic models of clean wing and flap-wing systems for the case of  $n = 8$ . The aeroelastic models are built using the doublet-hybrid method and using MSC/NASTRAN. For more than approximately 50% of  $q_F$ , the  $F_N$  parameter associated with a clean wing decreases almost linearly with respect to dynamic pressure. Additionally, for more than approximately 40% of  $q_F$ , the  $F_N$  parameter of a flap-wing decreases linearly with respect to dynamic pressure. The flutter dynamic pressure can be predicted by linear curve fitting of  $F_N$ . Figure 2 demonstrates that  $F_N$  decreases linearly against dynamic flutter for the multimode system. Therefore, the flutter speed can be predicted by extrapolating  $F_N$  independent of the number of modes.

#### Application to Subsonic Wind-Tunnel Test

In the preceding section, the behavior of the new flutter prediction parameter  $F_N$  has been numerically investigated. In this section, the parameter  $F_N$  is used to predict the flutter speeds of the two wind-tunnel test models shown in Fig. 1. The wings are excited by the flow disturbance, and the random signals of wing can be measured

**Table 1 Natural frequencies of clean wing and flap-wing**

Mode	Clean wing		Flap-wing	
	NASTRAN, Hz	Experiment, Hz	NASTRAN, Hz	Experiment, Hz
1	7.25	7.25	6.34	6.11
2	31.9	30.9	14.6	15.6
3	40.3	40.6	27.4	27.6
4	87.6	82.8	38.0	35.3

**Fig. 3 V-g plot, V-f plot, and  $F_N$  of clean wing.**

by a laser displacement sensor (Keyence LB-301). The subsonic wind tunnel has a speed range from about 8 m/s to about 50 m/s, and the sampling time is 3.9 ms. Before the wind-tunnel test, the free-vibration analysis and free-vibration tests for a clean wing and a flap-wing were performed to verify the finite element model. Table 1 provides a comparison of the predicted and experimentally obtained natural frequencies for these wings. For a clean wing, the displacement at the trailing edge of the wingtip is measured. For a flap-wing, the displacement at the trailing edge of the flap center is measured because of the measuring of the second flapping mode. The results obtained from MSC/NASTRAN are in good agreement with the experimental results. Using these finite element models and the flutter analysis method,<sup>11,12</sup> the flutter speeds are predicted numerically.

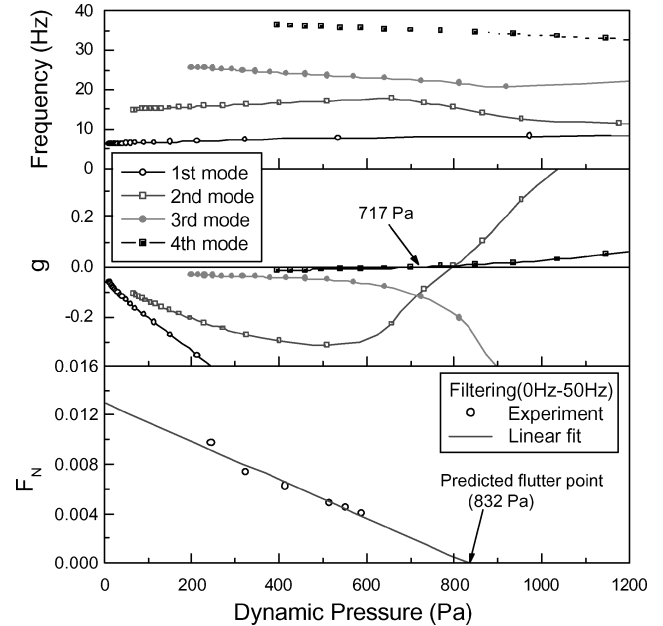
#### Flat-Plate Wing Model (Clean Wing)

From the measured signal, signal-processing methods such as the ARMA model estimation, flutter prediction parameter calculation, and flutter boundary prediction are conducted in an off-line analysis. To remove the effects of unmodeled modes, the digital data are processed using a digital filter to eliminate higher modes. The filtered data are then identified with an ARMA model by using the MATLAB System Identification Toolbox. The estimated coefficients of the ARMA model are then used to calculate the flutter prediction parameter  $F_N$  in Eq. (5). The parameters  $F_N$  are plotted against dynamic pressures  $q$ , and the flutter boundary can be predicted by an extrapolation of  $F_N$  vs  $q$  curve. From an aeroelastic module in MSC/NASTRAN<sup>11</sup> and  $V$ - $g$  (velocity and damping) (Ref. 12), the flutter speed is estimated to be about 43 m/s. The flutter mode shape is the first-second coalescent mode. Generally, the flutter mechanism of the flat plate wing is due to a coupling of the fundamental bending and torsion modes.

Figure 3 shows the  $V$ - $g$  plot,  $V$ - $f$  plot, and  $F_N$  of a clean wing. The wind-tunnel tests are conducted from about 20 to 40 m/s. From wind-tunnel test data, the flutter prediction parameter  $F_N$  with the

**Table 2 Flutter results**

Method	Speed, m/s	Frequency, Hz	Speed, m/s	Frequency, Hz
NASTRAN <sup>11</sup>	42.90	21.57	33.65	35.63
$V$ - $g$ <sup>12</sup>	42.84	20.83	34.20	35.09
Experiment	43.54		35.49	

**Fig. 4 V-g plot, V-f plot, and  $F_N$  of flap-wing.**

eighth-order ARMA model is calculated and shown in Fig. 3. The cutoff frequencies are fixed to 0 and 100 Hz for the eighth-order ARMA model. Figure 3 shows that the parameter  $F_N$  decreases almost linearly against the dynamic pressure. Hence, we know that the flutter prediction parameter  $F_N$  used in the present study has a linear behavior against a dynamic pressure independent of the number of modes and that the flutter boundary can be predicted by linear fitting of the  $F_N$  curve. As mentioned in Ref. 8,  $F_Z$  is sensitive to the filter bandwidth and the updated flutter parameter  $F_N$  is also sensitive to the filter bandwidth. The flutter speeds predicted through wind-tunnel tests and calculated by several flutter analysis methods<sup>11,12</sup> are shown in Table 2. The experimental predictions and numerical results for the clean-wing flutter agree well with each other.

#### Plate Wing Model with Flap (Flap-Wing)

The second model used in the wind-tunnel test is a plate wing with a flap, as shown in Fig. 1. Figure 4 shows the  $V$ - $g$  plot,  $V$ - $f$  plot, and  $F_N$  for a flap-wing. From Fig. 4, we know that two flutter speeds may exist and that these speeds are almost equal. One flutter speed is caused by the fourth mode (second bending mode, which corresponds to the hump-mode flutter<sup>12</sup>). The other flutter speed is due to the second-third coalescent mode. Thus, the flutter mechanism of the flap-wing is so complicated that at least four structural modes should be used to predict the flutter boundary experimentally and analytically. From the flutter analyses, the lowest flutter speed is found to be about 34 m/s, and the flutter frequency is about 35 Hz.

The wind-tunnel tests were conducted from about 20 to 30 m/s. The cutoff frequencies are fixed to 0 and 50 Hz, and the order of the ARMA model is eight. Figure 4 shows the flutter prediction parameter  $F_N$  estimated from wind-tunnel test data with the eighth-order ARMA model. As in Figs. 3 and 4, parameter  $F_N$  decreases almost linearly against dynamic pressure, and the flutter boundary can be predicted using a linear fitting. Table 2 provides a comparison of the experimentally predicted and numerical flutter speeds and shows that the results agree well with each other.

## Conclusions

A flutter prediction method to predict the flutter boundary from wind-tunnel test data has been investigated. The flutter prediction parameter  $F_Z$  proposed by Torii and Matsuzaki was updated to include multiple modes. Flutter analyses and the wind-tunnel tests of a flat-plate wing and a plate wing with flap were conducted. Numerical investigations of the flutter prediction parameter  $F_N$  used in the present study and wind-tunnel tests show that the new parameter  $F_N$  decreases almost linearly against dynamic pressure regardless of the order of the ARMA model. Therefore, the flutter boundary can be predicted using a linear curve fitting of  $F_N$ . The new flutter prediction parameter  $F_N$  that has been formulated in the present study can provide an accurate, safe, and reliable prediction of the flutter boundary and is applicable to a multimode flutter system.

## Acknowledgments

This research was supported by the Agency for Defense Development and was partially supported Ministry of Science and Technology (National Research Laboratory Program) in the Republic of Korea. This support is gratefully acknowledged. Authors express thanks to the associate editor Franklin Eastep, and to reviewers for many valuable comments and suggestions. Also, the authors appreciate the review and comment of Henry A. Sodano of Virginia Polytechnic Institute and State University about this paper.

## References

- <sup>1</sup>Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison-Wesley, Cambridge, MA, 1955, Chap. 1.
- <sup>2</sup>Dowell, E. H., Crawley, E. F., Curtiss, H. C., Jr., Peters, D. A., Scanlan, R. H., and Sisto, F., *A Modern Course in Aeroelasticity*, Kluwer Academic, Norwell, MA, 1995, Chap. 1.
- <sup>3</sup>Walker, R., and Gupta, N., "Real-Time Flutter Analysis," NASA CR-170412, March 1984.
- <sup>4</sup>Pak, C. G., and Friedmann, P. P., "New Time-Domain Technique for Flutter Boundary Identification," AIAA Paper 92-2102, April 1992.
- <sup>5</sup>Zimmermann, N. H., and Weissenburger, J. T., "Prediction of Flutter Onset Speed Based on Flight Testing at Subcritical Speeds," *Journal of Aircraft*, Vol. 1, No. 4, 1964, pp. 190–202.
- <sup>6</sup>Price, S. J., and Lee, B. H. K., "Evaluation and Extension of the Flutter Margin Method for Flight Flutter Prediction," *Journal of Aircraft*, Vol. 30, No. 3, 1993, pp. 395–402.
- <sup>7</sup>Matsuzaki, Y., and Ando, Y., "Estimation of Flutter Boundary from Random Responses due to Turbulence at Subcritical Speeds," *Journal of Aircraft*, Vol. 18, No. 10, 1981, pp. 862–868.
- <sup>8</sup>Torii, H., and Matsuzaki, Y., "Flutter Margin Evaluation for Discrete-Time Systems," *Journal of Aircraft*, Vol. 38, No. 1, 2001, pp. 42–47.
- <sup>9</sup>Lind, R., and Brenner, M., "Flutterometer: An On-Line Tool to Predict Robust Flutter Margins," *Journal of Aircraft*, Vol. 37, No. 6, 2000, pp. 1105–1112.
- <sup>10</sup>Jury, I. E., and Pavlidis, T., "Stability and Aperiodicity Constraints for System Design," *IEEE Transactions on Circuit Theory*, Vol. 10, No. 1, 1963, pp. 137–141.
- <sup>11</sup>Rodden, W. P., and Johnson, E. H., "MSC/NASTRAN Aeroelastic Analysis User's GUIDE," Ver. 68, MSC, Los Angeles, CA, 2002.
- <sup>12</sup>Bae, J. S., Yang, S. M., and Lee, I., "Linear and Nonlinear Aeroelastic Analysis of a Fighter-Type Wing with Control Surface," *Journal of Aircraft*, Vol. 30, No. 4, 2002, pp. 697–708.